Basic knowledge
Stability problem buckling

If slim and long components such as bars, beams and columns are subject to compressive stress owing to a force along the bar axis, these can end up in indifferent or unstable equilibrium positions. If the force $F$ is less than the critical force $F_K$, also known as buckling force, the component is in a stable equilibrium position and there is a strength problem. If the force $F$ reaches the buckling force $F_K$ of the bar, the bar suddenly starts to buckle. The components, thus, lose their ability to function. Buckling is usually a very sudden and abrupt process which causes large deformations.

Different equilibrium positions

<table>
<thead>
<tr>
<th>Stable equilibrium</th>
<th>Indifferent equilibrium</th>
<th>Unstable equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$ force</td>
<td>$F$ force</td>
<td>$F$ force</td>
</tr>
<tr>
<td>Bar returns to its starting position after the load is removed.</td>
<td>Bar remains in the new position after the load is removed.</td>
<td>Bar does not return to the starting position after the load is removed and does not stay in the position it assumed while the load is being applied. The bar falls over.</td>
</tr>
</tbody>
</table>

Stability in bars

Bars under pressure are a typical stability problem. Here, we investigate when a straight bar collapses. The critical buckling force $F_K$ describes the smallest possible compressive force under which the bar buckles. The critical buckling stress $\sigma_K$ is the stress that occurs at the critical buckling force $F_K$. The buckling force for pressure-loaded bars depends on the support conditions, bending stiffness and geometry of the shape of the bar cross-section. Euler’s four buckling cases are taken as the basis for the study of the bending stability of bars with constant bending stiffness.

Determining the buckling force $F_K$

$F_K = \frac{n^2 \cdot E \cdot I}{L K^2}$

Determining the buckling stress $\sigma_K$

$\sigma_K = \frac{n^2 \cdot E}{\lambda^2}$

$F$ force, $L$ bar length, $LK$ buckling length, $\beta$ buckling length coefficient.

Euler’s buckling cases

The mathematician and physicist Leonhard Euler defined four typical buckling cases to calculate the buckling force. For each of these cases, there is a buckling length coefficient $\beta$ that is used to determine the buckling length $L_K$.

Case 1: one bar end fixed, one bar end free
buckling length coefficient $\beta = 2$

Case 2: both bar ends pinned
buckling length coefficient $\beta = 1$

Case 3: one bar end fixed, one bar end pinned
buckling length coefficient $\beta = 0.7$

Case 4: both bar ends fixed
buckling length coefficient $\beta = 0.5$

$F$ force, $L$ bar length, $LK$ buckling length, $\beta$ buckling length coefficient.